



# Algebra 1

Summer Review Packet

for Students Entering

Algebra 2/Trigonometry

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Your Name

Mr. Sagman

of the key concepts and skills that you should have gained during a normal course in algebra 1. I have also included some topics that were covered during honors geometry as they will be revisited in algebra /trigonometry on a much higher level. It is to your advantage that you successfully complete this assignment if you wish any success in next year's honors mathematics course. If you need to relearn a certain topic, you should refer to an algebra 1 textbook. Most libraries have a mathematics section that you can consult. You may also choose to search online for any topic(s) of your choice.

Match each equation with the illustrated property

- |  |   |
|--|---|
| 1. $3(6y) = (3 \cdot 6)y$              | a. Identity Property of Multiplication    |
| 2. $23 + 25 + 7 = 23 + 7 + 25$         | b. Associative Property of Addition       |
| 3. $\frac{x}{3} + 0 = \frac{x}{3}$     | c. Commutative Property of Multiplication |
| 4. $(5y + 3x) + 10x = 5y + (3x + 10x)$ | d. Additive Inverse Property              |
| 5. $\frac{x}{y} \cdot \frac{y}{x} = 1$ | e. Distributive Property                  |
| 6. $v(3t) = (3t)v$                     | f. Commutative Property of Addition       |
| 7. $-26x + 26x = 0$                    | g. Multiplicative Inverse Property        |
| 8. $(3xyz) \cdot 1 = 3xyz$             | h. Associative Property of Multiplication |
| 9. $4(3x - 7) = 12x - 28$              | i. Additive Identity                      |

### Simplifying Expressions:

Simplify each expression. Write your answer in simplest form.

$$(2y^3 - 9y + 1) - (6y^3 + 3y - 3)$$

$$-7x + 8(-2x + 5)$$

$$4y(2 - y) + 3y^2$$

$$5(x + y) - 4(3x - 2y + 1)$$

$$\frac{30x^2 + 20x - 10}{-5}$$

$$(-3x^3 - 2x^2 + 5x + 4) + (-2x^3 + 7x - 6)$$

$$(3x^3 - 12x^2 - 5x + 1) - (x^3 - x^2 + 5x + 8)$$

# Adding and Subtracting Polynomials

**Add Polynomials** To add polynomials, you can group like terms horizontally or write them in column form, aligning like terms vertically. **Like terms** are monomial terms that are either identical or differ only in their coefficients, such as  $3p$  and  $-5p$  or  $2x^2y$  and  $8x^2y$ .

**Example 1** Find  $(2x^2 + x - 8) + (3x - 4x^2 + 2)$ .

## Horizontal Method

Group like terms.

$$(2x^2 + x - 8) + (3x - 4x^2 + 2)$$

$$= (2x^2 - 4x^2) + (x + 3x) + (-8 + 2) \\ = -2x^2 + 4x - 6.$$

The sum is  $-2x^2 + 4x - 6$ .

**Example 2** Find  $(3x^2 + 5xy) + (xy + 2x^2)$ .

## Vertical Method

Align like terms in columns and add.

$$\begin{array}{r} 3x^2 + 5xy \\ (+) \quad 2x^2 + \quad xy \\ \hline 5x^2 + 6xy \end{array}$$

The sum is  $5x^2 + 6xy$ .

## Exercises

Find each sum.

1.  $(4a - 5) + (3a + 6)$

2.  $(6x + 9) + (4x^2 - 7)$

3.  $(6xy + 2y + 6x) + (4xy - x)$

4.  $(x^2 + y^2) + (-x^2 + y^2)$

5.  $(3p^2 - 2p + 3) + (p^2 - 7p + 7)$

6.  $(2x^2 + 5xy + 4y^2) + (-xy - 6x^2 + 2y^2)$

7.  $(5p + 2q) + (2p^2 - 8q + 1)$

8.  $(4x^2 - x + 4) + (5x + 2x^2 + 2)$

9.  $(6x^2 + 3x) + (x^2 - 4x - 3)$

10.  $(x^2 + 2xy + y^2) + (x^2 - xy - 2y^2)$

11.  $(2a - 4b - c) + (-2a - b - 4c)$

12.  $(6xy^2 + 4xy) + (2xy - 10xy^2 + y^2)$

13.  $(2p - 5q) + (3p + 6q) + (p - q)$

14.  $(2x^2 - 6) + (5x^2 + 2) + (-x^2 - 7)$

15.  $(3z^2 + 5z) + (z^2 + 2z) + (z - 4)$

16.  $(8x^2 + 4x + 3y^2 + y) + (6x^2 - x + 4y)$

b.  $5(2-3)^2$

c.  $\frac{18-2\cdot 5}{15+3(-3)}$

### Solving Equations:

Solve each of the following equations for  $x$ .

$$3-2(x-1)=2+4x$$

$$8x-4+3(x+7)=6x-3(x-3)$$

$$16x-3(4x+7)=6x-(2x+21)$$

$$(x-3)-5(x+7)=1(x-3)-(7x+5)$$

$$-6x^2=-216$$

$$\frac{2}{3}=\frac{x+7}{3x}$$

$$\frac{x+6}{4}=\frac{-4x}{16}$$

$$16x+24=7(x+6)$$

$$5x+3=2x+18$$

$$2(x+1)-7=5$$

$$4(y+3)-2y=7$$

$$-10+2x+3(5-x)=\frac{1}{2}(4x-8)$$

$$5(y+2)-4(y-1)=6$$

$$3m+12=2(m-3)+4$$

$$\frac{x+1}{4}=5$$

$$\frac{x}{5}+\frac{x}{3}=10$$

# Linear Graphs:

Given two points M and N on the coordinate plane. Find the slope of  $\overline{MN}$  AND state the slope of the line perpendicular to  $\overline{MN}$ .

$$M(9,6), N(1,4)$$

$$M(-2,2), N(4,-4)$$

$$M(-9,16), N(-11,16)$$

Find the x-intercept and y-intercept of the given line. Using the intercepts, graph the line on the grid provided with the answer sheet.

$$y = x - 5$$

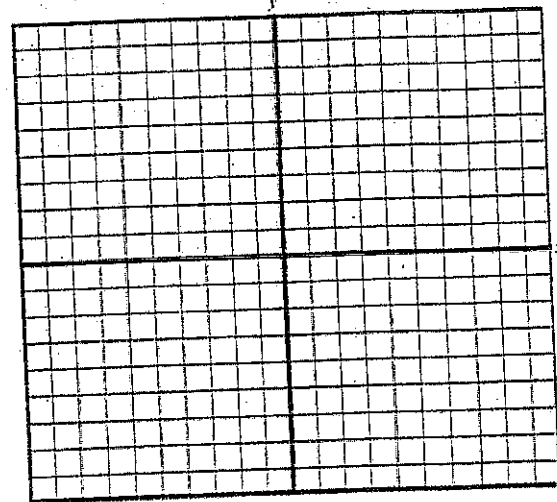
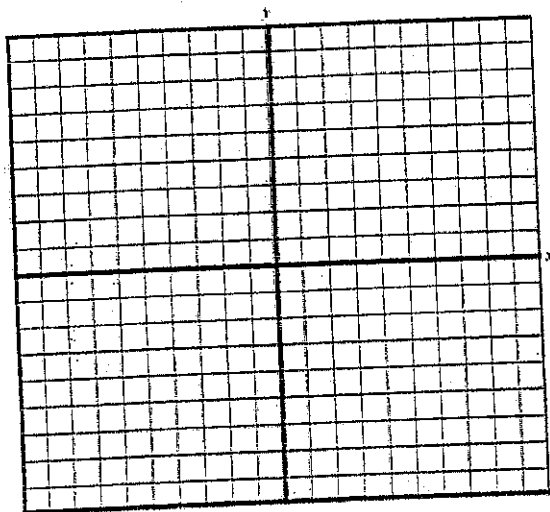
$$6x = 2y = -12$$

$$3y = 9x + 15$$

$$y = -2x + 1$$

$$y - 10 = 2(x - 4)$$

$$6x - 5 = 2y + 3$$

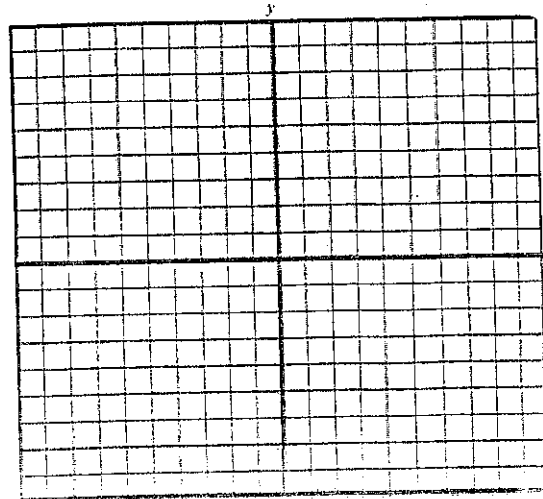
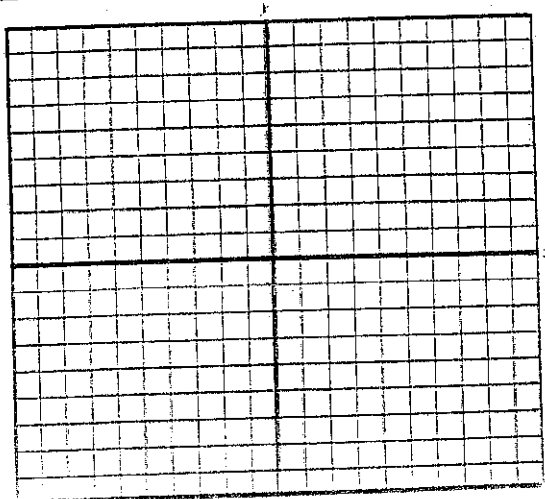


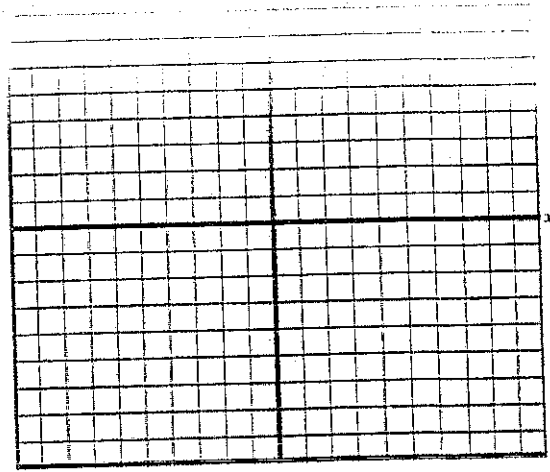
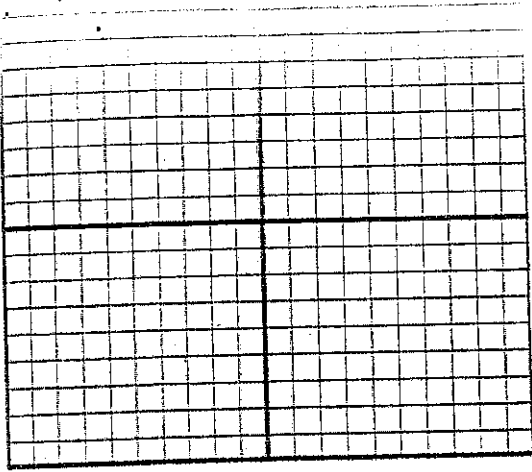
Intercepts:

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X: \_\_\_\_\_ Y: \_\_\_\_\_

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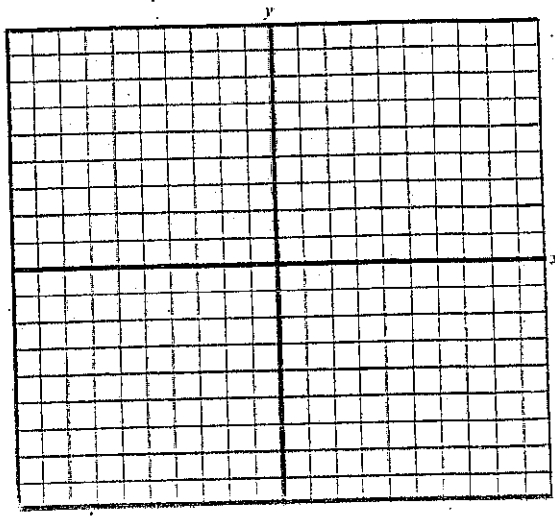
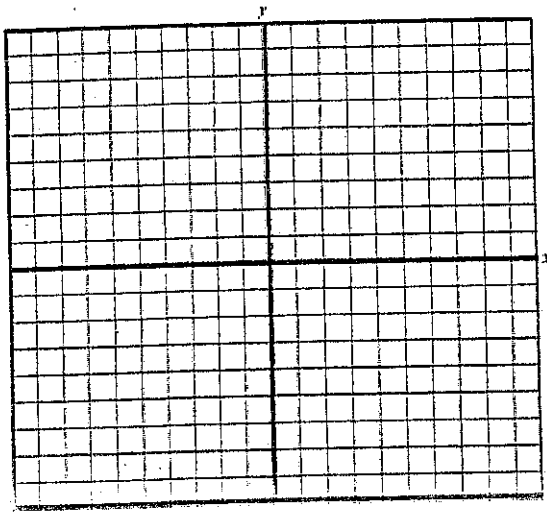
X: \_\_\_\_\_ Y: \_\_\_\_\_

X: \_\_\_\_\_ Y: \_\_\_\_\_

Find the slope and y-intercept of the graph of the equation. Using slope-intercept form, graph the line on the grid provided with the answer sheet.

$y - 2x = 7$

$3x + 6y = 12$



Slope: \_\_\_\_\_

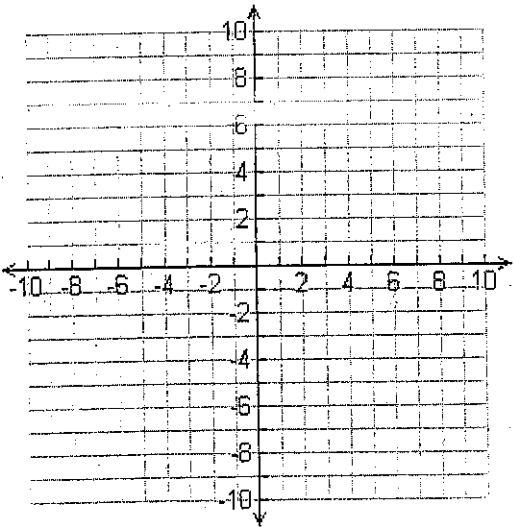
Slope: \_\_\_\_\_

Y-Intercept: \_\_\_\_\_

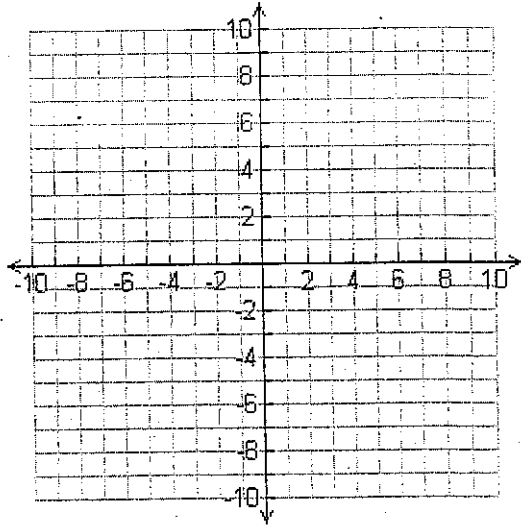
Y-Intercept: \_\_\_\_\_

Graph each line.

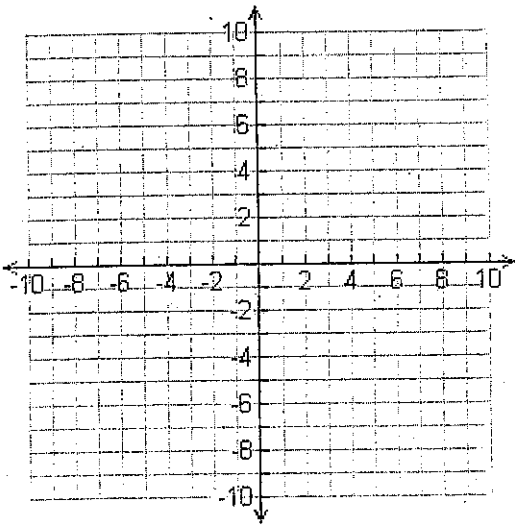
- a. The line whose x-intercept is 4 and y-intercept is -6



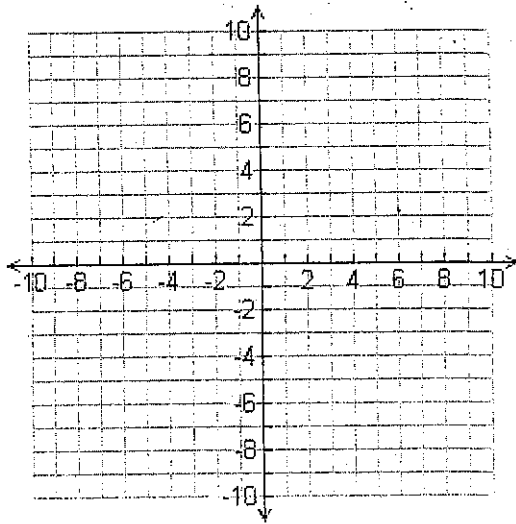
- b. The line that passes through the point  $(-1, 4)$  and a slope of  $-\frac{2}{3}$



c.  $y = 2x - 5$



d.  $2x + 4y = 8$



Write the equation of each line.

The line that has a slope of  $-\frac{3}{5}$  and passes through the point  $(0, -10)$ .

▪ Write it in slope-intercept form.

▪ Write it in standard form.

- slope =  $\frac{1}{2}$
- passes through  $(6,1)$  and  $(8, -5)$
- x-intercept =  $-3$ , y-intercept =  $6$
- passes through  $(4,2)$  and is parallel to the line whose equation is  $y = 2x - 4$ .
- passes through  $(-2,0)$  and is perpendicular to the line whose equation is  $y = -3x + 7$ .

An attorney charges a fee for each hour that she meets with a client. She charges \$250 for the first hour and \$150 for each additional hour.

- a. Make a table of the total charges for 1, 2, 3, and 4 total hours of meetings.
- b. Write a linear function to model the total charges vs the total hours of meetings.
- c. Find the total charges for 25 hours of meetings.



The line that passes through the points  $(-6, -6)$  and  $(-3, 1)$

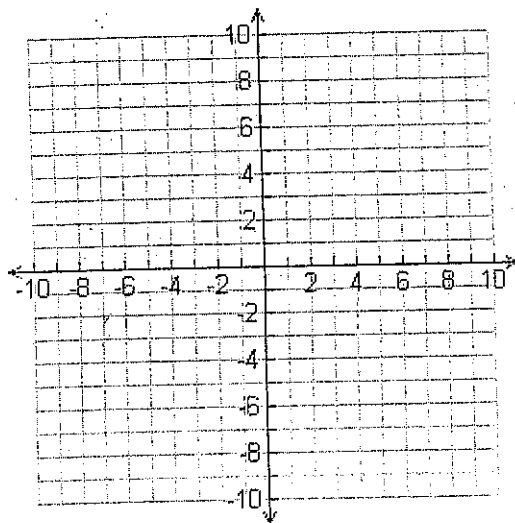
- Write it in point-slope form
- Write it in standard form

The line that is parallel to the line  $3x + 2y = 10$  and passes through the point  $(6, -3)$   
Write the final answer in standard form.

## Systems of Equations

Solve each system of equations using the indicated method.

a. 
$$\begin{cases} 2x - y = 1 \\ 3x + 2y = 12 \end{cases}$$
 Graphically



You can estimate the time,  $t$ , in hours that it takes to fly a distance,  $d$ , in miles by using the formula

$$t = \frac{d}{500} + \frac{1}{2}$$

- Use the formula to estimate the time it takes to fly 1300 miles.
- Solve the formula for  $d$
- Use the rewritten formula from part b to find how many miles you can fly in 4 hours.

# SUBSTITUTION

## Review:

Solve each of the following equations for  $y$  using the given value for  $x$ .

1.  $y = 3x + 2$  for  $x = -7$

2.  $y = -\frac{2}{5}x - 3$  for  $x = 10$

3.  $y = -2x + 9$  for  $x = y + 3$

To solve a system of equations using substitution:

Solve one equation for  $x$  (or  $y$ ).

Substitute this value into the other equation and solve for  $y$  (or  $x$ ).

Ex.  $y = 3x + 9$  and  $y - x = -7$

Harder Example  $7y = 2x + 63$  and  $2x - y = 15$

## Practice:

Solve each system using substitution.

1.  $y = -3x + 5$

$$x + y = 9$$

2.  $x - 2y = 5$

$$y = x - 6$$

3.  $y = -\frac{3}{2}x - 2$

$y = -2x + 1$

4.  $5x - 3y = -1$

$2x - y = 1$

## Elimination

To solve a system of equations using elimination:

Add the two equations to eliminate a variable (x or y).

Adjust the equations with multiplication before adding them if necessary.

Ex.  $4x + 5y = -12$  and

$3x - 5y = 26$

Harder Example:  $2x - 3y = -10$  and

$5x = 6y - 31$

### Practice:

Solve each system using elimination.

1.  $2x - y = -9$

$3x + y = -16$

2.  $4x - 3y = -17$

$2x + 5y = 11$

## Substitution and Elimination

Solve each using substitution or elimination.

$$3x + 4y = 2$$

$$4x = 4y + 12$$

$$y = 3x + 3$$

$$3x + 2y = -12$$

$$2x - 3y = -24$$

$$x + 6y = 18$$

$$y = -4x$$

$$x + 2y = -7$$

$$x = 3y - 4$$

$$2x + 6y = 5$$

$$3x - 2y = 11$$

$$x - \frac{1}{2}y = 4$$

$$0.3x - 0.2y = 0.5$$

$$x + 2y = 15$$

$$x - 7 = 2y$$

$$4x - y = 9$$

## Word Problems: Systems

### Word Problems Practice: Money problems.

Write a system of equations and solve:

1. Anna has a pocket of dimes and quarters. If she has 10 coins worth \$1.45, how many of her coins are quarters?
2. Popsicles cost \$0.80, and ice-cream cups cost \$0.65. If you purchased 9 items for \$6.15, how many of the items were popsicles.

### Word Problems Practice: Sum/Difference

Write a system of equations and solve:

1. The sum of two integers is 19 and their difference is 10. What is the smaller of the two integers?
2. If I add Mark's age to Tammy's age, I get 39. If I subtract Mark's age from Tammy's age, I get negative 7. What will I get if I multiply Mark's age by Tammy's?

### Word Problems Practice

Write a system of equations and solve:

1. Mr. Batterson ordered pizzas for the team. Medium pizzas have 8 slices and large pizzas have 10. If there are 13 pizzas and 108 slices, how many large pizza slices are there?
2. At a toy store, the children's department has bicycles and tricycles. There are 50 total, and 111 wheels. How many bicycles are there?

### Word Problems Practice: Time

Write a system of equations and solve:

1. In five years, Kate will be twice as old as Joey. Right now, Kate is 11 years older than Joey. How old is Joey right now?
2. A bucket is full of red marbles and white marbles. There are twice as many white marbles as red ones. If I add seven white marbles, there will be three times as many white marbles as red ones. How many marbles were in the bucket before the white marbles were added?

## HOW TO WORK WORD PROBLEMS IN ALGEBRA: PART II

### Word Problem Examples:

#### 1. Age:

Abigail is 6 years older than Jonathan. Six years ago she was twice as old as he. How old is each now?

**Solution:**

Let  $x$  = Jonathan's age now (smaller number)  
 $x + 6$  = Abigail's age now  
 $x - 6$  = Jonathan's age 6 years ago  
 $(x + 6) - 6$  = Abigail's age 6 years ago

Six years ago she was twice as old as he.

**Equation:**

$$\begin{aligned}(x + 6) - 6 &= 2(x - 6) \\ x &= 2x - 12 \\ -x &= -12\end{aligned}$$

**Answers:**  $x = 12$  (Jonathan's age now)  
 $x + 6 = 18$  (Abigail's age now)

#### 2. Numbers:

There are two numbers whose sum is 72. One number is twice the other. What are the numbers?

**Steps:**

1. Read the problem. It is about numbers.
2. The question at the end asks, "What are the numbers?" So we start out "let  $x$  = smaller number." Be sure you always start with  $x$  (that is,  $1x$ ). Never start off with "let  $2x$  = something," because it is meaningless unless you know what  $x$  stands for.
3. Now let's put it all together and solve.

**Solution:**

Let  $x$  = smaller number  
 $2x$  = larger number

$$\begin{aligned}\text{Then } x + 2x &= 72 \\ 3x &= 72\end{aligned}$$

**Answers:**  $x = 24$  (smaller number)  
 $2x = 48$  (larger number)

**Check:** The sum of the numbers is 72. Thus  $24 + 48 = 72$ .

### 3. Rate, Time, and Distance:

A freight train starts from Los Angeles and heads for Chicago at 40 mph. Two hours later a passenger train leaves the same station for Chicago traveling 60 mph. How long before the passenger train overtakes the freight train?

**Steps:**

1. Read the problem through carefully.
2. The question at the end of the problem asks "how long?" (which means time) for the passenger train. **Time** is your unknown.

**Solution:**

Draw sketch of movement.

|           | Rate | Time | Distance |
|-----------|------|------|----------|
| Freight   |      |      |          |
| Passenger |      |      |          |

Now read the problem again from the beginning for those individual steps. The freight train traveled 40 mph, and "Two hours later a passenger train... traveling 60 mph":

|           | Rate | Time | Distance |
|-----------|------|------|----------|
| Freight   | 40   |      |          |
| Passenger | 60   |      |          |

But so far you haven't any unknown. The question asked is, "How long before the passenger train overtakes the freight train?"

Let  $x$  = time in hours for the passenger train.

The passenger train started 2 hours after the freight train, so the freight train took 2 hours longer. You can represent the time for the freight train by  $x + 2$ :

|           | Rate | Time    | Distance |
|-----------|------|---------|----------|
| Freight   | 40   | $x + 2$ |          |
| Passenger | 60   | $x$     |          |

Now, rate times time equals distance ( $r \times t = d$ ), so multiply what you have in the rate box times what you have in the time box, and put the result in the distance box:

|           | Rate | Time    | Distance    |
|-----------|------|---------|-------------|
| Freight   | 40   | $x + 2$ | $40(x + 2)$ |
| Passenger | 60   | $x$     | $60x$       |

Set these two distances equal for your equation:

$$\begin{aligned}
 40(x + 2) &= 60x \\
 40x + 80 &= 60x \\
 -20x &= -80
 \end{aligned}$$

Answer:  $x = 4$

Check:  $40(4 + 2) = 60(4)$   
 $40(6) = 240$   
 $240 = 240$

#### 4. Mixtures:

A mixture containing 6% boric acid is to be mixed with 2 quarts of a mixture which is 15% boric acid in order to obtain a solution which is 12% boric acid. How much of the 6% solution must be used?

##### Steps:

1. Draw three diagrams to represent the three solutions.

|         |   |         |   |         |
|---------|---|---------|---|---------|
| 1st     |   | 2nd     |   | Total   |
| Mixture | + | Mixture | = | Mixture |

2. Find out what the percents are in each mixture and the amount and put it in the proper place:

|     |   |             |   |       |
|-----|---|-------------|---|-------|
| 1st |   | 2nd         |   | Total |
| 6%  | + | 2 qt<br>15% | = | 12%   |

3. The question asks, "How much of the 6% solution must be used?" Therefore, let  $x$  represent the amount of 6% solution used. The total solution will have  $x$  quarts plus 2 quarts (or the total of the other two solutions).

Let  $x$  = quarts of 6% solution

|        |   |      |   |              |
|--------|---|------|---|--------------|
| $x$ qt |   | 2 qt |   | $(x + 2)$ qt |
| 6%     | + | 15%  | = | 12%          |

You now are ready to solve the problem. If you multiply the amount of solution by the percent of acid in the solution, you will find the amount of pure boric acid in each solution. The amount of pure acid in the final solution is equal to the sum of the amounts of pure acid in the two original solutions.

|           |  |           |  |               |
|-----------|--|-----------|--|---------------|
| $0.06x$   |  | $2(0.15)$ |  | $0.12(x + 2)$ |
| Pure acid |  | Pure acid |  | Pure acid     |

Equation:  $0.06x + 2(0.15) = 0.12(x + 2)$

Eliminate parentheses first:  $0.06x + 0.30 = 0.12x + 0.24$

To clear decimals, multiply by 100 which moves all decimal points two places to the right:

$$6x + 30 = 12x + 24$$
$$-6x = -6$$

Answer:  $x = 1$

Check:  $0.06x + 0.30 = 0.12x + 0.24$   
 $0.06(1) + 0.30 = 0.12(1) + 0.24$   
 $0.36 = 0.36$

#### 5. Coins:

Michael has some coins in his pocket consisting of dimes, nickels, and pennies. He has two more nickels than dimes, and three times as many pennies as nickels. How many of each kind of coins does he have if the total value is 52 cents?

##### Steps:

1. Determine which of the coin he has the fewest. This is often a good way to find what  $x$  represent. Here he has fewer dimes than nickels or pennies.



2. The question asks **how many** of each kind of coin does he have. (Not how much they are worth!). That is, what **number** of each kind of coin does he have? So, let  $x$  = number of dimes.
3. Look at one fact at a time. He has two more nickels than dimes. Let

$$x + 2 = \text{number of nickels}$$

4. Next fact, he has "three times as many pennies as nickels."

$$3(x + 2) = \text{number of pennies}$$

| Number of Coins                | Value in Cents                          |
|--------------------------------|---|
| $x$ = number of dimes          | $10x$ = number of cents in dimes        |
| $x + 2$ = number of nickels    | $5(x + 2)$ = number of cents in nickels |
| $3(x + 2)$ = number of pennies | $3(x + 2)$ = number of cents in pennies |

Now add the **amounts of money**. If you make it all pennies, there are no decimals.

$$10x + 5(x + 2) + 3(x + 2) = 52$$

$$10x + 5x + 10 + 3x + 6 = 52$$

$$8x = 36$$

- Answers:**  $x = 2$  (number of dimes)  
 $x + 2 = 4$  (number of nickels)  
 $3(x + 2) = 12$  (number of pennies)

**Check:**  $2(10) = 20$  cents in dimes  
 $4(5) = 20$  cents in nickels  
 $12(1) = 12$  cents in pennies

$$\text{Total} = 52 \text{ cents}$$

### FACTS TO REMEMBER ABOUT SOLVING AN EQUATION

These are facts you should have already learned about procedures in problem solutions.

1. Remove parentheses first.

$$\text{subtraction } -(3x + 2) = -3x - 2$$

$$\text{multiplication (distributive law) } 3(x + 2) = 3x + 6$$

2. Remove fractions by multiplying by the lowest common denominator.

$$3/x + 4 = 5/2x + 3$$

The LCD is  $2x$ . Multiplying both sides of the equation by  $2x$ :

$$6 + 8x = 5 + 6x$$

3. Decimals **MAY** be removed from an equation before solving. Multiply by a power of 10 large enough to make all decimal numbers whole numbers.

**Example:**

$$0.03x + 201.2 - x = 85$$

Multiply both sides of the equation by 100.

$$3x + 20120 - 100x = 8500$$

### Word Problem Practice:

1. Think of a number. Double the number. Subtract 6 from the result and divide the answer by 2. The quotient will be 20. What is the number?
2. There are three consecutive even numbers such that twice the first is 20 more than the second. Find the numbers.
3. Jay's father is twice as old as Jay. In 20 years Jay will be two-thirds as old as his father. How old is each now?
4. Wolfgang and Heinrich worked as electricians at \$14 and \$12 per hour respectively. One month Wolfgang worked 10 hours more than Heinrich. If their total income for the month was \$3520, how many hours did each work during the month?
5. Three-fifths of the men in chemistry class have beards and two-thirds of the women have long hair. If there are 120 men in the class and 46 are not in the above groups, how many men and how many women are there in the class?
6. A service station checks Mr. Gittleboro's radiator and finds it contains only 30% antifreeze. If the radiator holds 10 quarts and is full, how much must be drained off and replaced with pure antifreeze in order to bring it up to a required 50% antifreeze?
7. Tickets for the baseball games were \$2.50 for general admission and 50 cents for kids. If there were six times as many general admissions sold as there were kids' tickets, and total receipts were \$7750, how many of each type of ticket were sold?
8. Bob has a coin collection made up of pennies and nickels. If he has three times as many pennies as nickels and the total face value of the coins is \$416, how many coins of each kind are in the collection?
9. The Allisons are on a cross-country trip traveling with the Jensions. One day they get separated and the Jensions are 20 miles ahead of the Allisons on the same road. If the Jensions average 50 mph and the Allisons travel at 60 mph, how long will it be before the Allisons catch up with the Jensions?
10. A reservoir can be filled by an inlet pipe in 24 hours and emptied by an outlet pipe in 28 hours. The foreman starts to fill the reservoir, but he forgets to close the outlet pipe. Six hours later he remembers and closes the outlet. How long does it take altogether to fill the reservoir?

#### Answers:

- |  |                             |
|--|-----------------------------|
| 1. 23                                      | 6. $2\frac{6}{7}$ quarts    |
| 2. 22, 24, 26                              | 7. 6500, 3000               |
| 3. 20, 40                                  | 8. 200 nickels, 600 pennies |
| 4. 130, 140                                | 9. 2 hours                  |
| 5. 90 men, 30 women, $29\frac{1}{7}$ hours | 10. $29\frac{1}{7}$ hours   |

# RADICALS

Perform the indicated operation and simplify the final answer!

$$2\sqrt{5} + 4\sqrt{5} - 3\sqrt{5}$$

$$\sqrt{72} \cdot \sqrt{42}$$

$$\sqrt{92} \cdot \sqrt{115}$$

$$\sqrt{12} + 4\sqrt{15}$$

$$\sqrt{192}$$

$$(2\sqrt{40})(3\sqrt{60})$$

$$\sqrt{288}(3\sqrt{108})$$

$$\sqrt{6} \cdot \sqrt{15}$$

$$\sqrt{\frac{9}{64}}$$

Simplify each expression

a.  $\sqrt{12}$

b.  $\sqrt{45}$

c.  $\sqrt{75}$

d.  $2\sqrt{3} + 4\sqrt{3}$

e.  $3\sqrt{5} - \sqrt{20}$

f.  $(4\sqrt{2})(3\sqrt{2})$

g.  $\frac{2\sqrt{6}}{3\sqrt{18}}$

$$\sqrt{52} + \sqrt{117}$$

$$\sqrt{175} - \sqrt{252}$$

$$\sqrt{1584}$$

$$5\sqrt{60} + 2\sqrt{135}$$

$$2\sqrt{80} - 3\sqrt{45} + 3\sqrt{245}$$

$$\sqrt{2645}$$

c)  $3\sqrt{7} + 8\sqrt{7}$

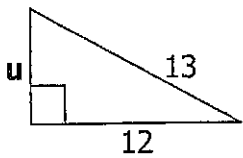
d)  $5\sqrt{24} - 4\sqrt{6}$

e)  $\frac{5}{\sqrt{3}}$

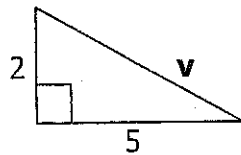
f)  $\sqrt{5}(5 + \sqrt{5})$

Find the missing side length for each triangle (show your steps):

a)



b)



Simplify each radical.

1.  $\sqrt{121}$

2.  $\sqrt{90}$

3.  $\sqrt{175}$

4.  $\sqrt{288}$

5.  $\sqrt{486}$

6.  $2\sqrt{16}$

7.  $6\sqrt{500}$

8.  $3\sqrt{147}$

9.  $8\sqrt{475}$

10.  $\sqrt{\frac{125}{9}}$

## Special Products

**Product of a Sum and a Difference** There is also a pattern for the product of a sum and a difference of the same two terms,  $(a + b)(a - b)$ . The product is called the **difference of squares**.

|                                   |                              |
|-----------------------------------|------------------------------|
| Product of a Sum and a Difference | $(a + b)(a - b) = a^2 - b^2$ |
|-----------------------------------|------------------------------|

**Example** Find  $(5x + 3y)(5x - 3y)$ .

$$\begin{aligned}(a + b)(a - b) &= a^2 - b^2 && \text{Product of a Sum and a Difference} \\(5x + 3y)(5x - 3y) &= (5x)^2 - (3y)^2 && a = 5x \text{ and } b = 3y \\ &= 25x^2 - 9y^2 && \text{Simplify.}\end{aligned}$$

The product is  $25x^2 - 9y^2$ .

### Exercises

Find each product.

1.  $(x - 4)(x + 4)$

2.  $(p + 2)(p - 2)$

3.  $(4x - 5)(4x + 5)$

4.  $(2x - 1)(2x + 1)$

5.  $(h + 7)(h - 7)$

6.  $(m - 5)(m + 5)$

7.  $(2c - 3)(2c + 3)$

8.  $(3 - 5q)(3 + 5q)$

9.  $(x - y)(x + y)$

10.  $(y - 4x)(y + 4x)$

11.  $(8 + 4x)(8 - 4x)$

12.  $(3a - 2b)(3a + 2b)$

13.  $(3y - 8)(3y + 8)$

14.  $(x^2 - 1)(x^2 + 1)$

15.  $(m^2 - 5)(m^2 + 5)$

16.  $(x^3 - 2)(x^3 + 2)$

17.  $(h^2 - k^2)(h^2 + k^2)$

18.  $\left(\frac{1}{4}x + 2\right)\left(\frac{1}{4}x - 2\right)$

19.  $(3x - 2y^2)(3x + 2y^2)$

20.  $(2p - 5s)(2p + 5s)$

21.  $\left(\frac{4}{3}x - 2y\right)\left(\frac{4}{3}x + 2y\right)$

c.  $(3x - 2)(2x - 5)$

d.  $(x + 6)(x - 6)$

e.  $(2x - 1)^2$

f.  $(4x - 3)(x^2 + 2x - 5)$

Factor each polynomial

a.  $x^2 + 8x + 15$

b.  $x^2 - 4x - 21$

c.  $x^2 - 9x + 20$

d.  $x^2 - 6x + 9$

e.  $x^2 - 16$

f.  $2x^2 - 5x - 3$

g.  $3x^2 + 5x + 2$

h.  $5x^2 - 14x - 3$

i.  $4x^2 + 11x + 6$

j.  $6x^2 + 17x + 10$

$3x - 21$

$x^2 + 6x + 8$

$2x^2 - 22x + 60$

$x^2 + 13x + 40$

Algebra Worksheet  
Special Factoring

Factor Completely

1.  $x^2 - 25$

2.  $x^2 - 100$

3.  $9x^2 - 1$

4.  $64x^2 - 9$

5.  $36y^2 + 49$

6.  $x^2 - 6x + 9$

7.  $y^2 + 14y + 49$

8.  $4a^2 - 20a + 25$

9.  $2x^2 - 72$

10.  $4x^2 - 16$

11.  $3x^2 + 18x - 27$

12.  $5x^2 + 10x + 5$

13.  $9 - x^2$

14.  $75 - 3x^2$

Solve each equation by factoring

15.  $x^2 - 64 = 0$

16.  $16x^2 - 9 = 0$

17.  $x^2 - 12x + 36 = 0$

18.  $4b^2 + 28b + 49 = 0$

19.  $50x^2 = 98x$

20.  $27a^3 + 18a^2 = -3a$

Factor

1.  $4x^2 + 6x + 2$

2.  $12x^2 + 33x - 9$

3.  $30y^2 + 10y - 20$

4.  $18a^2 - 24a + 6$

5.  $14a^2 - 21a + 7$

6.  $7x^2 + 50x + 7$

7.  $2x^3 - 11x^2 + 5x$

8.  $3a^3 - 16a^2 + 16a$

9.  $6x^3 - 11x^2 - 10x$

10.  $6p^3 + 5p^2 + p$

11.  $3x^3 + 3x^2 - 36x$

12.  $2x^3 - 2x^2 - 4x$

13.  $p^4 + 9p^3 - 36p^2$

14.  $6p^4 - 32p^3 + 5p^2$

Solve each equation by factoring

15.  $15y^2 - 50y + 35 = 0$

16.  $8x^2 + 4x - 4 = 0$

17.  $2x^3 - 3x^2 - 5x = 0$

18.  $10x^3 + 5x^2 - 5x = 0$

19.  $-6 = 4y^2 - 11y$

20.  $-14a^2 = 12a^3 - 48a$



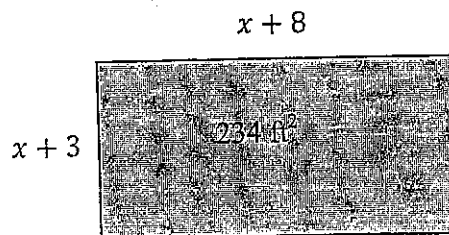
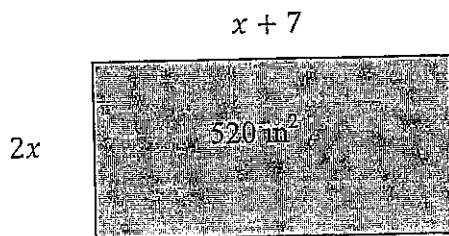
**Practice solving quadratics by factoring.**

1.  $x^2 + 5x + 6 = 0$
3.  $a^2 - 9a + 18 = 0$
5.  $x^2 + 15x + 30 = -6$
7.  $2x^2 + 6x + 4 = 0$
9.  $c^2 - 6c + 9 = 0$
11.  $h^2 - 7 = 9$
13.  $d^2 + 10d + 18 = -7$
15.  $11a^2 - 32a - 15 = 0$
17.  $5x^2 - 11x - 3 = 2x + 3$
19.  $12h^2 + 40h + 32 = 0$

2.  $x^2 - x - 12 = 0$
4.  $t^2 + 2t - 19 = 5$
6.  $d^2 + 10d = -16$
8.  $3a^2 - 12a = 15$
10.  $5x^2 - 14x + 8 = 0$
12.  $7t^2 - 15t + 6 = 4$
14.  $4x^2 - 46 = 3$
16.  $4n^2 + 12n + 9 = 0$
18.  $6t^2 - 15t - 36 = 0$

**Challenge Problems**

20.  $3x^3 + 21x^2 + 36x = 0$
22.  $x^4 - 13x^2 + 36 = 0$
24. Find the dimensions of the rectangle below.
21.  $2a^3 - 18a^2 + 36a = 0$
23.  $x^4 + 3x^2 - 4 = 0$
25. Find the dimensions of the rectangle below.



## Dividing Monomials

**Quotients of Monomials** To divide two powers with the same base, subtract the exponents.

|                     |  |
|---------------------|--|
| Quotient of Powers  | For all integers $m$ and $n$ and any nonzero number $a$ , $\frac{a^m}{a^n} = a^{m-n}$ .                              |
| Power of a Quotient | For any integer $m$ and any real numbers $a$ and $b$ , $b \neq 0$ , $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$ . |

**Example 1** Simplify  $\frac{a^4b^7}{ab^2}$ . Assume neither  $a$  nor  $b$  is equal to zero.

$$\begin{aligned} \frac{a^4b^7}{ab^2} &= \left(\frac{a^4}{a}\right)\left(\frac{b^7}{b^2}\right) && \text{Group powers with the same base.} \\ &= (a^{4-1})(b^{7-2}) && \text{Quotient of Powers} \\ &= a^3b^5 && \text{Simplify.} \end{aligned}$$

The quotient is  $a^3b^5$ .

**Example 2** Simplify  $\left(\frac{2a^3b^5}{3b^2}\right)^3$ .

Assume that  $b$  is not equal to zero.

$$\begin{aligned} \left(\frac{2a^3b^5}{3b^2}\right)^3 &= \frac{(2a^3b^5)^3}{(3b^2)^3} && \text{Power of a Quotient} \\ &= \frac{2^3(a^3)^3(b^5)^3}{(3)^3(b^2)^3} && \text{Power of a Product} \\ &= \frac{8a^9b^{15}}{27b^6} && \text{Power of a Power} \\ &= \frac{8a^9b^9}{27} && \text{Quotient of Powers} \end{aligned}$$

The quotient is  $\frac{8a^9b^9}{27}$ .

### Exercises

Simplify. Assume that no denominator is equal to zero.

1.  $\frac{5^5}{5^2}$

2.  $\frac{m^8}{m^4}$

3.  $\frac{p^5n^4}{p^2n}$

4.  $\frac{a^2}{a}$

5.  $\frac{x^5y^3}{x^5y^2}$

6.  $\frac{-2y^7}{14y^5}$

7.  $\frac{xy^6}{y^4x}$

8.  $\left(\frac{2a^2b}{a}\right)^3$

9.  $\left(\frac{4p^4q^4}{3p^2q^2}\right)^3$

# Dividing Monomials

**Negative Exponents** Any nonzero number raised to the zero power is 1; for example,  $(-0.5)^0 = 1$ . Any nonzero number raised to a negative power is equal to the reciprocal of the number raised to the opposite power; for example,  $6^{-3} = \frac{1}{6^3}$ . These definitions can be used to simplify expressions that have negative exponents.

|                                   |  |
|-----------------------------------|--|
| <b>Zero Exponent</b>              | For any nonzero number $a$ , $a^0 = 1$ .   |
| <b>Negative Exponent Property</b> | For any nonzero number $a$ and any integer $n$ , $a^{-n} = \frac{1}{a^n}$ and $\frac{1}{a^{-n}} = a^n$ . |

The simplified form of an expression containing negative exponents must contain only positive exponents.

## Example

Simplify  $\frac{4a^{-3}b^6}{16a^2b^6c^{-5}}$ . Assume that the denominator is not equal to zero.

$$\frac{4a^{-3}b^6}{16a^2b^6c^{-5}} = \left(\frac{4}{16}\right)\left(\frac{a^{-3}}{a^2}\right)\left(\frac{b^6}{b^6}\right)\left(\frac{1}{c^{-5}}\right)$$

Group powers with the same base.

$$= \frac{1}{4}(a^{-3-2})(b^{6-6})(c^5)$$

Quotient of Powers and Negative Exponent Properties

$$= \frac{1}{4}a^{-5}b^0c^5$$

Simplify.

$$= \frac{1}{4}\left(\frac{1}{a^5}\right)(1)c^5$$

Negative Exponent and Zero Exponent Properties

$$= \frac{c^5}{4a^5}$$

Simplify.

The solution is  $\frac{c^5}{4a^5}$ .

## Exercises

Simplify. Assume that no denominator is equal to zero.

1.  $\frac{2^2}{2^{-3}}$

2.  $\frac{m}{m^{-4}}$

3.  $\frac{p^{-8}}{p^3}$

4.  $\frac{b^{-4}}{b^{-5}}$

5.  $\frac{(-x^{-1}y)^0}{4w^{-1}y^2}$

6.  $\frac{(a^2b^3)^2}{(ab)^{-2}}$

7.  $\frac{x^4y^0}{x^{-2}}$

8.  $\frac{(6a^{-1}b)^2}{(b^2)^4}$

9.  $\frac{(3st)^2u^{-4}}{s^{-1}t^2u^7}$

10.  $\frac{s^{-3}t^{-5}}{(s^2t^3)^{-1}}$

11.  $\left(\frac{4m^2n^2}{8m^{-1}l}\right)^0$

12.  $\frac{(-2mn^2)^{-3}}{4m^{-6}n^4}$

Solve and graph each inequality.

1.  $4d + 7 \leq 23$

2.  $-7 \leq 5 - 4a$

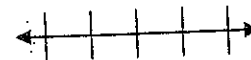
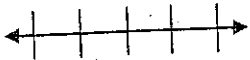
3.  $2(j - 4) \geq -6$



4.  $-3 > 3(5x - 16)$

5.  $3t + 7 > 5t + 9$

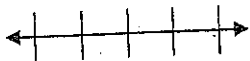
6.  $-3(v - 3) < 5 - 4v$



7.  $\frac{4}{3}r - 3 < r + \frac{2}{3} - \frac{1}{3}r$

8.  $4p - 5 + p > -7 + 5p + 2$

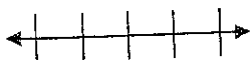
9.  $-3(v - 3) < 5 - 4v$



10.  $2n - 3(n + 3) \leq 14$

11.  $9j + 3 \leq 3(3j + 1)$

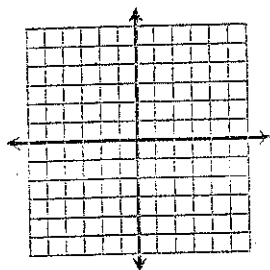
12.  $5x - \frac{1}{2}(3x + 8) \leq -4 + 3x$



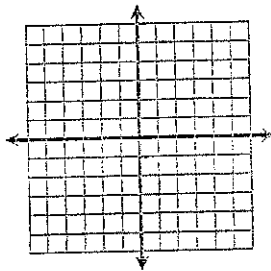
Solving Systems of Inequalities by Graphing

Solve each system of inequalities by graphing.

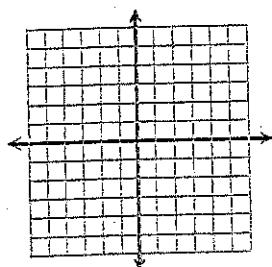
1. 
$$\begin{cases} y > -x + 4 \\ y \leq x + 2 \end{cases}$$



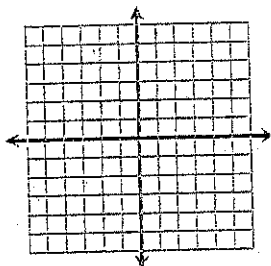
2. 
$$\begin{cases} y < -\frac{1}{2}x - 3 \\ y \geq \frac{2}{3}x - 3 \end{cases}$$



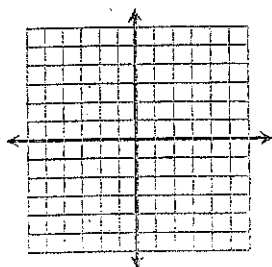
3. 
$$\begin{cases} y + 3 \leq 2x \\ x \leq 3 \end{cases}$$



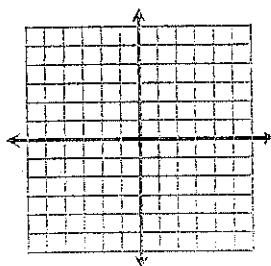
4. 
$$\begin{cases} y \geq x + 1 \\ y - x > -2 \end{cases}$$



5. 
$$\begin{cases} y < -2x + 4 \\ y < 3 \end{cases}$$



6. 
$$\begin{cases} y - x \leq 4 \\ -2x + y \geq 4 \end{cases}$$



Name: \_\_\_\_\_

Algebra 1

Worksheet: Absolute Value Equations 2

Nov. 14, 2012

Solve these absolute value equations:

- First get the absolute value term by itself
- Make 2 equations (+ & -) w/o absolute value
- Solve each equation to get 2 solutions
- Check your solutions by plugging into the original absolute value equation

|                             |                              |
|-----------------------------|------------------------------|
| <u>Example:</u>             | $ 2x + 5  - 3 = 8$           |
| Isolate the absolute value: | $ 2x + 5  = 11$              |
| Make 2 equations:           | $2x + 5 = 11$ $2x + 5 = -11$ |
| Solve each equation:        | $2x = 6$ $2x = -16$          |
|                             | $x = 3$ $x = -8$             |

1.  $|x + 4| = 10$

2.  $|x - 1.8| = 3.2$

3.  $|3x - 1| = 14$

4.  $|2y + 7| = 7$

5.  $|y - 8| - 2 = 5$

6.  $|x + 17| + 10 = 70$

7.  $|x - 5| + 6 = 0$

8.  $|x - 9| - 13 = -8$

9.  $|5x + 2| - 4 = 18$

10.  $|4 - y| = 7$

$$11. 3|x + 7| - 14 = 4$$

$$12. -2|y - 7| + 18 = 8$$

$$13. 2|6 - 2x| - 1 = 7$$

$$14. -5|4y - 11| - 3 = 12$$

$$15. \frac{1}{5}|x - 5| - 4 = 2$$

$$16. \frac{3}{4}|y + 2| = 6$$

Challenge:

$$17. |x + 3| = 3x + 1$$

Solving Radical Equations

1.  $\sqrt{2x-5} = 5$

2.  $7 - \sqrt{2x+2} = 3$

3.  $2(x+1)^2 = 54$

4.  $\sqrt{3x+4} + x = 8$

5.  $\sqrt{3a+1} = a-1$

6.  $\sqrt{x+5} = x+3$

7.  $2 + \sqrt{x} = \sqrt{2x+7}$

8.  $4 - 5(2y-7)^2 = -11$

9.  $\sqrt{x} + \sqrt{x-8} = 4$

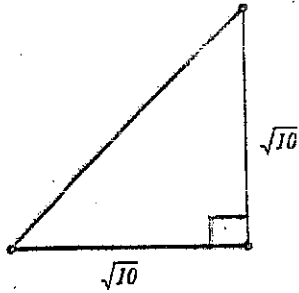


Homework – Worksheet – 45-45-90 Triangles

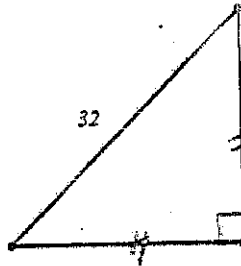
Name: \_\_\_\_\_

1. Find the length of each missing side:

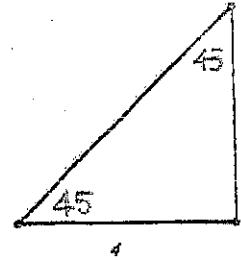
a.



b.



c.



2. Find the perimeter of a square with diagonal length 20.

3. Find the diagonal of a square with perimeter  $8\sqrt{2}$

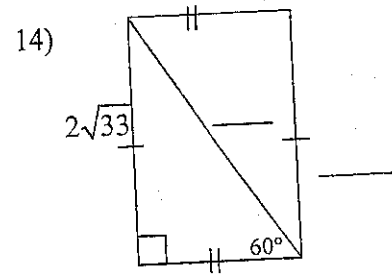
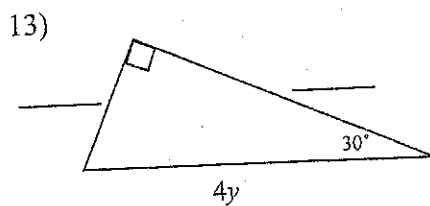
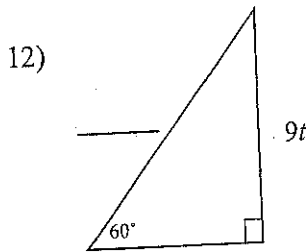
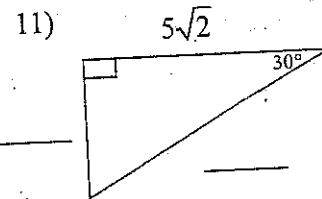
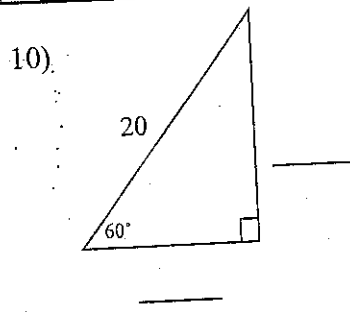
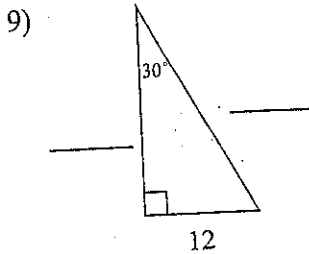
4. The two diagonals of a square create four isosceles right triangles. If the side length of the square is  $4\sqrt{6}$  find the side lengths of each small right triangle.

1) In a 30°-60°-90° triangle, the short leg is located across from what angle?

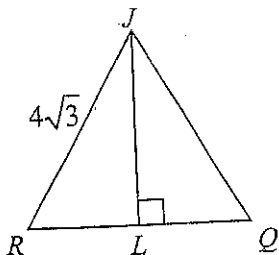
Complete the table for a 30°-60°-90° triangle using exact (radical) values.

|        | Short Leg     | Long Leg    | Hypotenuse    |
|--------|---------------|-------------|---------------|
| Ratios | $x$           |             |               |
| 2)     | 5             |             |               |
| 3)     |               |             | 14            |
| 4)     |               | $6\sqrt{3}$ |               |
| 5)     | $2\sqrt{3}$   |             |               |
| 6)     |               | 9           |               |
| 7)     |               |             | $10y\sqrt{3}$ |
| 8)     | $7ab\sqrt{2}$ |             |               |

Fill in the blanks for the special right triangles.

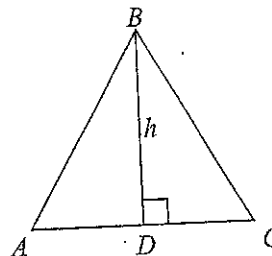


15)  $\triangle RJQ$  is equilateral.



- $JQ =$  \_\_\_\_\_  
 $RL =$  \_\_\_\_\_  
 $LQ =$  \_\_\_\_\_  
 $JL =$  \_\_\_\_\_

16)  $\triangle ABC$  is equilateral.



- $AD =$  \_\_\_\_\_  
 $DC =$  \_\_\_\_\_  
 $AB =$  \_\_\_\_\_  
 $BC =$  \_\_\_\_\_

Identify if the given values could be the sides of a 30°-60°-90° triangle.

17)  $2, 2\sqrt{3}, 4$

18)  $9, 3, 3\sqrt{3}$

19)  $\sqrt{3}, 3, \sqrt{6}$

20)  $4\sqrt{6}, 2\sqrt{6}, 6\sqrt{2}$

21) The hypotenuse of a 30-60-90 triangle is 20m. Find the length of the side opposite the 30° angle.

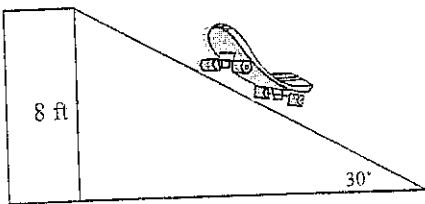
22) The hypotenuse of a 30-60-90 triangle is  $12\sqrt{2}$  ft. Find the area of the triangle.

23) Find the perimeter and area of a 30°-60°-90° triangle with hypotenuse length 28 centimeters.

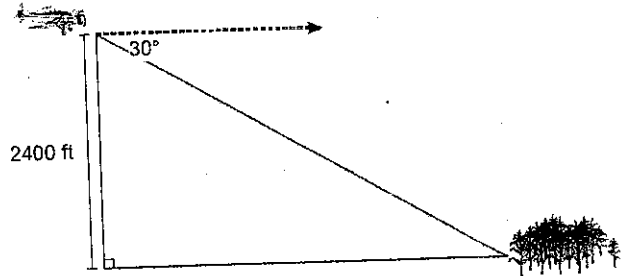
24) Find the perimeter and area of an equilateral triangle with side length 4 feet.

25) Find the perimeter and area of an equilateral triangle with height 30 yards.

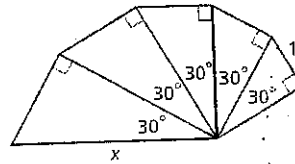
26) A skate board ramp must be set up to rise from the ground at 30°. If the height from the ground to the platform is 8 feet, how far away from the platform must the ramp be set?



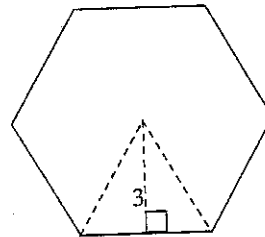
27) Mr. Bridgewater is flying his single-engine plane at an altitude of 2400 feet. He sees a cornfield at an angle of depression of 30°. What is his approximate horizontal distance from the cornfield at this point?



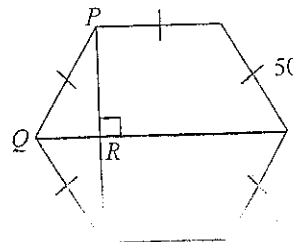
28) Find the value of  $x$  in simplest radical form.



29) Find the perimeter of the regular hexagon below. Answer in simplest radical form.



30) Find  $QR$  and  $PS$ . Answer in simplest radical form.



Set 1 (do these in June)

Set 2 (do these in July)

|  |   |
|--|---|
| 1. Evaluate $4(2+3 \cdot 5) - 3^2$ ; use order of operations.                              | 1. Subtract: $(7x^3 - 5x + 2) - (5x^3 - 4x^2 + 6x - 7)$ |
| 2. If $x=3$ and $y=-7$ ; find the value of $3x-5y$ .                                       | 2. Multiply: $6x^2(5x-3)$                               |
| 3. Simplify $35-7(3z-2)$ .   | 3. Multiply out: $(5a-3)(2a+4)$                         |
| 4. Write 0.32 as a fraction in lowest terms.   | 4. Simplify: $\frac{10x^5y^4}{15x^3y^9}$                |
| 5. Find the y-intercept of $6x-y=-12$  | 5. Find the product: $(x-3)^2$                          |
| 6. Determine the equation of the line with slope of -3 and containing the point $(-7,2)$ . | 6. Find the product: $(a-4)(a+4)$                       |
| 7. Find the slope of the line containing the points $(-1, 2)$ and $(5, -4)$ .              | 7. Factor: $x^2 - 7x - 30$                              |
| 8. Write the equation of the vertical line that contains the point $(-5, -4)$ .            | 8. Factor: $x^2 + 6x - 16$                              |
| 9. Find the slope of the line $x-2y=6$   | 9. Factor: $2x^2 - 11x + 5$                             |
| 10. Solve: $\frac{3}{2}x + 4 = -9$   | 10. Factor: $16a^2 - 25b^2$                             |
| 11. Solve: $2(3x-7) + 4x = 26$   | 11. Solve by factoring: $x^2 - x - 12 = 0$              |
| 12. Solve: $4-3x=5-6x-7$   | 12. Solve: $(x+6)(x-7)(x-8)(x+9) = 0$                   |
| 13. Solve and graph on a number line: $5-3x < 14$  | 13. Simplify: $\sqrt{75}$                               |
| 14. Solve by cross multiplying: $\frac{x}{x+2} = \frac{3}{7}$                              | 14. Simplify: $\frac{6x}{5y} \cdot \frac{10y}{8x}$      |
| 15. Solve the system: $3x-2y=10$<br>$y=2x+5$   | 15. Simplify: $\frac{3x}{x+4} - \frac{x+5}{x+4}$        |
| 16. Solve the system: $6x-3y=11$<br>$6x+3y=17$   | 16. Simplify: $\sqrt{\frac{5}{3}}$                      |
| 17. Solve the system: $3x+5y=22$<br>$4x+3y=11$   | 17. Simplify: $\sqrt{180}$                              |
| 18. Simplify: $(3x^2)(-2x^5)$  | 18. Simplify: $7\sqrt{28} + 3\sqrt{63}$                 |
| 19. Simplify: $(5a^2b^3)^2$  | 19. Simplify: $\frac{9x^2 + 27}{-3}$                    |
| 20. Simplify: $(4a^3)^2(3a)^2$   | 20. Solve by the quadratic formula: $2x^2 - 3x - 1 = 0$ |